10.4.1 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 10 materials.

In Exercises 1 - 6, use the Even / Odd Identities to verify the identity. Assume all quantities are defined.

1. $\sin(3\pi - 2\theta) = -\sin(2\theta - 3\pi)$
2. $\cos\left(-\frac{\pi}{4} - 5t\right) = \cos\left(5t + \frac{\pi}{4}\right)$
3. $\tan(-t^2 + 1) = -\tan(t^2 - 1)$
4. $\csc(-\theta - 5) = -\csc(\theta + 5)$
5. $\sec(-6t) = \sec(6t)$
6. $\cot(9 - 7\theta) = -\cot(7\theta - 9)$

In Exercises 7 - 21, use the Sum and Difference Identities to find the exact value. You may have need of the Quotient, Reciprocal or Even / Odd Identities as well.

For help with these exercises, click one or more of the resources below:

- Using the Sum and Difference Identities
- Using the Quotient, Reciprocal, and Pythagorean Identities

7. $\cos(75^\circ)$
8. $\sec(165^\circ)$
9. $\sin(105^\circ)$
10. $\csc(195^\circ)$
11. $\cot(255^\circ)$
12. $\tan(375^\circ)$
13. $\cos\left(\frac{13\pi}{12}\right)$
14. $\sin\left(\frac{11\pi}{12}\right)$
15. $\tan\left(\frac{13\pi}{12}\right)$
16. $\cos\left(\frac{7\pi}{12}\right)$
17. $\tan\left(\frac{17\pi}{12}\right)$
18. $\sin\left(\frac{\pi}{12}\right)$
19. $\cot\left(\frac{11\pi}{12}\right)$
20. $\csc\left(\frac{5\pi}{12}\right)$
21. $\sec\left(-\frac{\pi}{12}\right)$
22. If α is a Quadrant IV angle with $\cos(\alpha) = \frac{\sqrt{5}}{5}$, and $\sin(\beta) = \frac{\sqrt{10}}{10}$, where $\frac{\pi}{2} < \beta < \pi$, find
    (a) $\cos(\alpha + \beta)$
    (b) $\sin(\alpha + \beta)$
    (c) $\tan(\alpha + \beta)$
    (d) $\cos(\alpha - \beta)$
    (e) $\sin(\alpha - \beta)$
    (f) $\tan(\alpha - \beta)$

23. If $\csc(\alpha) = 3$, where $0 < \alpha < \frac{\pi}{2}$, and β is a Quadrant II angle with $\tan(\beta) = -7$, find

    (a) $\cos(\alpha + \beta)$ (b) $\sin(\alpha + \beta)$ (c) $\tan(\alpha + \beta)$

    (d) $\cos(\alpha - \beta)$ (e) $\sin(\alpha - \beta)$ (f) $\tan(\alpha - \beta)$

24. If $\sin(\alpha) = \frac{3}{5}$, where $0 < \alpha < \frac{\pi}{2}$, and $\cos(\beta) = \frac{12}{13}$ where $\frac{3\pi}{2} < \beta < 2\pi$, find

    (a) $\sin(\alpha + \beta)$ (b) $\cos(\alpha - \beta)$ (c) $\tan(\alpha - \beta)$

25. If $\sec(\alpha) = -\frac{5}{3}$, where $\frac{\pi}{2} < \alpha < \pi$, and $\tan(\beta) = \frac{24}{7}$, where $\pi < \beta < \frac{3\pi}{2}$, find

    (a) $\csc(\alpha - \beta)$ (b) $\sec(\alpha + \beta)$ (c) $\cot(\alpha + \beta)$

In Exercises 26 - 38, verify the identity.

For help with these exercises, click one or more of the resources below:

- Using the Sum and Difference Identities
- Using the Quotient, Reciprocal, and Pythagorean Identities

26. $\cos(\theta - \pi) = -\cos(\theta)$

27. $\sin(\pi - \theta) = \sin(\theta)$

28. $\tan\left(\theta + \frac{\pi}{2}\right) = -\cot(\theta)$

29. $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin(\alpha)\cos(\beta)$

30. $\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2\cos(\alpha)\sin(\beta)$

31. $\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2\cos(\alpha)\cos(\beta)$

32. $\cos(\alpha + \beta) - \cos(\alpha - \beta) = -2\sin(\alpha)\sin(\beta)$

33. $\dfrac{\sin(\alpha + \beta)}{\sin(\alpha - \beta)} = \dfrac{1 + \cot(\alpha)\tan(\beta)}{1 - \cot(\alpha)\tan(\beta)}$

34. $\dfrac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \dfrac{1 - \tan(\alpha)\tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$

35. $\dfrac{\tan(\alpha + \beta)}{\tan(\alpha - \beta)} = \dfrac{\sin(\alpha)\cos(\alpha) + \sin(\beta)\cos(\beta)}{\sin(\alpha)\cos(\alpha) - \sin(\beta)\cos(\beta)}$

36. $\dfrac{\sin(t + h) - \sin(t)}{h} = \cos(t)\left(\dfrac{\sin(h)}{h}\right) + \sin(t)\left(\dfrac{\cos(h) - 1}{h}\right)$

37. $\dfrac{\cos(t + h) - \cos(t)}{h} = \cos(t)\left(\dfrac{\cos(h) - 1}{h}\right) - \sin(t)\left(\dfrac{\sin(h)}{h}\right)$

38. $\dfrac{\tan(t + h) - \tan(t)}{h} = \left(\dfrac{\tan(h)}{h}\right)\left(\dfrac{\sec^2(t)}{1 - \tan(t)\tan(h)}\right)$

In Exercises 39 - 48, use the Half Angle Formulas to find the exact value. You may have need of the Quotient, Reciprocal or Even / Odd Identities as well.

For help with these exercises, click one or more of the resources below:

- Using the Half Angle Identities
- Using the Quotient, Reciprocal, and Pythagorean Identities

39. $\cos(75^\circ)$ (compare with Exercise 7)
40. $\sin(105^\circ)$ (compare with Exercise 9)
41. $\cos(67.5^\circ)$
42. $\sin(157.5^\circ)$
43. $\tan(112.5^\circ)$
44. $\cos\left(\dfrac{7\pi}{12}\right)$ (compare with Exercise 16)
45. $\sin\left(\dfrac{\pi}{12}\right)$ (compare with Exercise 18)
46. $\cos\left(\dfrac{\pi}{8}\right)$
47. $\sin\left(\dfrac{5\pi}{8}\right)$
48. $\tan\left(\dfrac{7\pi}{8}\right)$

In Exercises 49 - 58, use the given information about θ to find the exact values of

- $\sin(2\theta)$
- $\cos(2\theta)$
- $\tan(2\theta)$
- $\sin\left(\dfrac{\theta}{2}\right)$
- $\cos\left(\dfrac{\theta}{2}\right)$
- $\tan\left(\dfrac{\theta}{2}\right)$

For help with these exercises, click one or more of the links below:

- Using the Double Angle Identities
- Using the Half Angle Identities
- Using the Reciprocal, Quotient, and Pythagorean Identities

49. $\sin(\theta) = -\dfrac{7}{25}$ where $\dfrac{3\pi}{2} < \theta < 2\pi$
50. $\cos(\theta) = \dfrac{28}{53}$ where $0 < \theta < \dfrac{\pi}{2}$
51. $\tan(\theta) = \dfrac{12}{5}$ where $\pi < \theta < \dfrac{3\pi}{2}$
52. $\csc(\theta) = 4$ where $\dfrac{\pi}{2} < \theta < \pi$
53. $\cos(\theta) = \dfrac{3}{5}$ where $0 < \theta < \dfrac{\pi}{2}$
54. $\sin(\theta) = -\dfrac{4}{5}$ where $\pi < \theta < \dfrac{3\pi}{2}$
55. $\cos(\theta) = \dfrac{12}{13}$ where $\dfrac{3\pi}{2} < \theta < 2\pi$
56. $\sin(\theta) = \dfrac{5}{13}$ where $\dfrac{\pi}{2} < \theta < \pi$

57. $\sec(\theta) = \sqrt{5}$ where $\frac{3\pi}{2} < \theta < 2\pi$

58. $\tan(\theta) = -2$ where $\frac{\pi}{2} < \theta < \pi$

In Exercises 59 - 73, verify the identity. Assume all quantities are defined.

For help with these exercises, click one or more of the links below:

- Using the Double Angle Identities
- Using the Half Angle Identities
- Using the Reciprocal, Quotient, and Pythagorean Identities

59. $(\cos(\theta) + \sin(\theta))^2 = 1 + \sin(2\theta)$

60. $(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$

61. $\tan(2\theta) = \dfrac{1}{1 - \tan(\theta)} - \dfrac{1}{1 + \tan(\theta)}$

62. $\csc(2\theta) = \dfrac{\cot(\theta) + \tan(\theta)}{2}$

63. $8\sin^4(\theta) = \cos(4\theta) - 4\cos(2\theta) + 3$

64. $8\cos^4(\theta) = \cos(4\theta) + 4\cos(2\theta) + 3$

65. $\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$

66. $\sin(4\theta) = 4\sin(\theta)\cos^3(\theta) - 4\sin^3(\theta)\cos(\theta)$

67. $32\sin^2(\theta)\cos^4(\theta) = 2 + \cos(2\theta) - 2\cos(4\theta) - \cos(6\theta)$

68. $32\sin^4(\theta)\cos^2(\theta) = 2 - \cos(2\theta) - 2\cos(4\theta) + \cos(6\theta)$

69. $\cos(4\theta) = 8\cos^4(\theta) - 8\cos^2(\theta) + 1$

70. $\cos(8\theta) = 128\cos^8(\theta) - 256\cos^6(\theta) + 160\cos^4(\theta) - 32\cos^2(\theta) + 1$ (HINT: Use the result to 69.)

71. $\sec(2\theta) = \dfrac{\cos(\theta)}{\cos(\theta) + \sin(\theta)} + \dfrac{\sin(\theta)}{\cos(\theta) - \sin(\theta)}$

72. $\dfrac{1}{\cos(\theta) - \sin(\theta)} + \dfrac{1}{\cos(\theta) + \sin(\theta)} = \dfrac{2\cos(\theta)}{\cos(2\theta)}$

73. $\dfrac{1}{\cos(\theta) - \sin(\theta)} - \dfrac{1}{\cos(\theta) + \sin(\theta)} = \dfrac{2\sin(\theta)}{\cos(2\theta)}$

In Exercises 74 - 79, write the given product as a sum. You may need to use an Even/Odd Identity.

74. $\cos(3\theta)\cos(5\theta)$

75. $\sin(2\theta)\sin(7\theta)$

76. $\sin(9\theta)\cos(\theta)$

77. $\cos(2\theta)\cos(6\theta)$

78. $\sin(3\theta)\sin(2\theta)$

79. $\cos(\theta)\sin(3\theta)$

In Exercises 80 - 85, write the given sum as a product. You may need to use an Even/Odd or Cofunction Identity.

80. $\cos(3\theta) + \cos(5\theta)$
81. $\sin(2\theta) - \sin(7\theta)$
82. $\cos(5\theta) - \cos(6\theta)$
83. $\sin(9\theta) - \sin(-\theta)$
84. $\sin(\theta) + \cos(\theta)$
85. $\cos(\theta) - \sin(\theta)$
86. Suppose θ is a Quadrant I angle with $\sin(\theta) = x$. Verify the following formulas

    (a) $\cos(\theta) = \sqrt{1 - x^2}$
    (b) $\sin(2\theta) = 2x\sqrt{1 - x^2}$
    (c) $\cos(2\theta) = 1 - 2x^2$

87. Discuss with your classmates how each of the formulas, if any, in Exercise 86 change if we change assume θ is a Quadrant II, III, or IV angle.
88. Suppose θ is a Quadrant I angle with $\tan(\theta) = x$. Verify the following formulas

    (a) $\cos(\theta) = \dfrac{1}{\sqrt{x^2 + 1}}$
    (b) $\sin(\theta) = \dfrac{x}{\sqrt{x^2 + 1}}$
    (c) $\sin(2\theta) = \dfrac{2x}{x^2 + 1}$
    (d) $\cos(2\theta) = \dfrac{1 - x^2}{x^2 + 1}$

89. Discuss with your classmates how each of the formulas, if any, in Exercise 88 change if we change assume θ is a Quadrant II, III, or IV angle.
90. If $\sin(\theta) = \dfrac{x}{2}$ for $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, find an expression for $\cos(2\theta)$ in terms of x.
91. If $\tan(\theta) = \dfrac{x}{7}$ for $-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$, find an expression for $\sin(2\theta)$ in terms of x.
92. If $\sec(\theta) = \dfrac{x}{4}$ for $0 < \theta < \dfrac{\pi}{2}$, find an expression for $\ln|\sec(\theta) + \tan(\theta)|$ in terms of x.
93. Show that $\cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$ for all θ.
94. Let θ be a Quadrant III angle with $\cos(\theta) = -\dfrac{1}{5}$. Show that this is not enough information to determine the sign of $\sin\left(\dfrac{\theta}{2}\right)$ by first assuming $3\pi < \theta < \dfrac{7\pi}{2}$ and then assuming $\pi < \theta < \dfrac{3\pi}{2}$ and computing $\sin\left(\dfrac{\theta}{2}\right)$ in both cases.
95. Without using your calculator, show that $\dfrac{\sqrt{2 + \sqrt{3}}}{2} = \dfrac{\sqrt{6} + \sqrt{2}}{4}$
96. In part 4 of Example 10.4.3, we wrote $\cos(3\theta)$ as a polynomial in terms of $\cos(\theta)$. In Exercise 69, we had you verify an identity which expresses $\cos(4\theta)$ as a polynomial in terms of $\cos(\theta)$. Can you find a polynomial in terms of $\cos(\theta)$ for $\cos(5\theta)$? $\cos(6\theta)$? Can you find a pattern so that $\cos(n\theta)$ could be written as a polynomial in cosine for any natural number n?

97. In Exercise 65, we has you verify an identity which expresses $\sin(3\theta)$ as a polynomial in terms of $\sin(\theta)$. Can you do the same for $\sin(5\theta)$? What about for $\sin(4\theta)$? If not, what goes wrong?

98. Verify the Even / Odd Identities for tangent, secant, cosecant and cotangent.

99. Verify the Cofunction Identities for tangent, secant, cosecant and cotangent.

100. Verify the Difference Identities for sine and tangent.

101. Verify the Product to Sum Identities.

102. Verify the Sum to Product Identities.

Checkpoint Quiz 10.4

1. Suppose $\pi < \theta < \frac{3\pi}{2}$ with $\cos(\theta) = -\frac{2\sqrt{5}}{5}$.
   (a) Find $\sin(\theta)$
   (b) Find $\cos(2\theta)$
   (c) Find $\sin(2\theta)$
   (d) In which Quadrant does the terminal side 2θ lie, when plotted in standard position?
   (e) Find $\cos\left(\frac{\theta}{2}\right)$
   (f) Find $\sin\left(\frac{\theta}{2}\right)$

2. Suppose α is a Quadrant II angle with $\sin(\alpha) = \frac{3}{5}$ and β is a Quadrant III angle with $\tan(\beta) = 5$. Find the six trigonometric functions of $(\alpha + \beta)$.

3. Verify the identity: $\sin(\theta) = \dfrac{2\tan(\theta/2)}{1 + \tan^2(\theta/2)}$

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2
- Quiz Solution Part 3

10.4.2 Answers

7. $\cos(75^\circ) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

8. $\sec(165^\circ) = -\dfrac{4}{\sqrt{2}+\sqrt{6}} = \sqrt{2}-\sqrt{6}$

9. $\sin(105^\circ) = \dfrac{\sqrt{6}+\sqrt{2}}{4}$

10. $\csc(195^\circ) = \dfrac{4}{\sqrt{2}-\sqrt{6}} = -(\sqrt{2}+\sqrt{6})$

11. $\cot(255^\circ) = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

12. $\tan(375^\circ) = \dfrac{3-\sqrt{3}}{3+\sqrt{3}} = 2-\sqrt{3}$

13. $\cos\left(\dfrac{13\pi}{12}\right) = -\dfrac{\sqrt{6}+\sqrt{2}}{4}$

14. $\sin\left(\dfrac{11\pi}{12}\right) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

15. $\tan\left(\dfrac{13\pi}{12}\right) = \dfrac{3-\sqrt{3}}{3+\sqrt{3}} = 2-\sqrt{3}$

16. $\cos\left(\dfrac{7\pi}{12}\right) = \dfrac{\sqrt{2}-\sqrt{6}}{4}$

17. $\tan\left(\dfrac{17\pi}{12}\right) = 2+\sqrt{3}$

18. $\sin\left(\dfrac{\pi}{12}\right) = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

19. $\cot\left(\dfrac{11\pi}{12}\right) = -(2+\sqrt{3})$

20. $\csc\left(\dfrac{5\pi}{12}\right) = \sqrt{6}-\sqrt{2}$

21. $\sec\left(-\dfrac{\pi}{12}\right) = \sqrt{6}-\sqrt{2}$

22. (a) $\cos(\alpha+\beta) = -\dfrac{\sqrt{2}}{10}$ (b) $\sin(\alpha+\beta) = \dfrac{7\sqrt{2}}{10}$

(c) $\tan(\alpha+\beta) = -7$ (d) $\cos(\alpha-\beta) = -\dfrac{\sqrt{2}}{2}$

(e) $\sin(\alpha-\beta) = \dfrac{\sqrt{2}}{2}$ (f) $\tan(\alpha-\beta) = -1$

23. (a) $\cos(\alpha+\beta) = -\dfrac{4+7\sqrt{2}}{30}$ (b) $\sin(\alpha+\beta) = \dfrac{28-\sqrt{2}}{30}$

(c) $\tan(\alpha+\beta) = \dfrac{-28+\sqrt{2}}{4+7\sqrt{2}} = \dfrac{63-100\sqrt{2}}{41}$ (d) $\cos(\alpha-\beta) = \dfrac{-4+7\sqrt{2}}{30}$

(e) $\sin(\alpha-\beta) = -\dfrac{28+\sqrt{2}}{30}$ (f) $\tan(\alpha-\beta) = \dfrac{28+\sqrt{2}}{4-7\sqrt{2}} = -\dfrac{63+100\sqrt{2}}{41}$

24. (a) $\sin(\alpha+\beta) = \dfrac{16}{65}$ (b) $\cos(\alpha-\beta) = \dfrac{33}{65}$ (c) $\tan(\alpha-\beta) = \dfrac{56}{33}$

25. (a) $\csc(\alpha - \beta) = -\dfrac{5}{4}$ (b) $\sec(\alpha + \beta) = \dfrac{125}{117}$ (c) $\cot(\alpha + \beta) = \dfrac{117}{44}$

39. $\cos(75^\circ) = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

40. $\sin(105^\circ) = \dfrac{\sqrt{2+\sqrt{3}}}{2}$

41. $\cos(67.5^\circ) = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

42. $\sin(157.5^\circ) = \dfrac{\sqrt{2-\sqrt{2}}}{2}$

43. $\tan(112.5^\circ) = -\sqrt{\dfrac{2+\sqrt{2}}{2-\sqrt{2}}} = -1-\sqrt{2}$

44. $\cos\left(\dfrac{7\pi}{12}\right) = -\dfrac{\sqrt{2-\sqrt{3}}}{2}$

45. $\sin\left(\dfrac{\pi}{12}\right) = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

46. $\cos\left(\dfrac{\pi}{8}\right) = \dfrac{\sqrt{2+\sqrt{2}}}{2}$

47. $\sin\left(\dfrac{5\pi}{8}\right) = \dfrac{\sqrt{2+\sqrt{2}}}{2}$

48. $\tan\left(\dfrac{7\pi}{8}\right) = -\sqrt{\dfrac{2-\sqrt{2}}{2+\sqrt{2}}} = 1-\sqrt{2}$

49.
- $\sin(2\theta) = -\dfrac{336}{625}$
- $\cos(2\theta) = \dfrac{527}{625}$
- $\tan(2\theta) = -\dfrac{336}{527}$
- $\sin\left(\frac{\theta}{2}\right) = \dfrac{\sqrt{2}}{10}$
- $\cos\left(\frac{\theta}{2}\right) = -\dfrac{7\sqrt{2}}{10}$
- $\tan\left(\frac{\theta}{2}\right) = -\dfrac{1}{7}$

50.
- $\sin(2\theta) = \dfrac{2520}{2809}$
- $\cos(2\theta) = -\dfrac{1241}{2809}$
- $\tan(2\theta) = -\dfrac{2520}{1241}$
- $\sin\left(\frac{\theta}{2}\right) = \dfrac{5\sqrt{106}}{106}$
- $\cos\left(\frac{\theta}{2}\right) = \dfrac{9\sqrt{106}}{106}$
- $\tan\left(\frac{\theta}{2}\right) = \dfrac{5}{9}$

51.
- $\sin(2\theta) = \dfrac{120}{169}$
- $\cos(2\theta) = -\dfrac{119}{169}$
- $\tan(2\theta) = -\dfrac{120}{119}$
- $\sin\left(\frac{\theta}{2}\right) = \dfrac{3\sqrt{13}}{13}$
- $\cos\left(\frac{\theta}{2}\right) = -\dfrac{2\sqrt{13}}{13}$
- $\tan\left(\frac{\theta}{2}\right) = -\dfrac{3}{2}$

52.
- $\sin(2\theta) = -\dfrac{\sqrt{15}}{8}$
- $\cos(2\theta) = \dfrac{7}{8}$
- $\tan(2\theta) = -\dfrac{\sqrt{15}}{7}$
- $\sin\left(\frac{\theta}{2}\right) = \dfrac{\sqrt{8+2\sqrt{15}}}{4}$
- $\cos\left(\frac{\theta}{2}\right) = \dfrac{\sqrt{8-2\sqrt{15}}}{4}$
- $\tan\left(\frac{\theta}{2}\right) = \sqrt{\dfrac{8+2\sqrt{15}}{8-2\sqrt{15}}}$
  $\tan\left(\frac{\theta}{2}\right) = 4+\sqrt{15}$

53.
- $\sin(2\theta) = \dfrac{24}{25}$
- $\cos(2\theta) = -\dfrac{7}{25}$
- $\tan(2\theta) = -\dfrac{24}{7}$
- $\sin\left(\frac{\theta}{2}\right) = \dfrac{\sqrt{5}}{5}$
- $\cos\left(\frac{\theta}{2}\right) = \dfrac{2\sqrt{5}}{5}$
- $\tan\left(\frac{\theta}{2}\right) = \dfrac{1}{2}$

54.
- $\sin(2\theta) = \frac{24}{25}$
- $\cos(2\theta) = -\frac{7}{25}$
- $\tan(2\theta) = -\frac{24}{7}$
- $\sin\left(\frac{\theta}{2}\right) = \frac{2\sqrt{5}}{5}$
- $\cos\left(\frac{\theta}{2}\right) = -\frac{\sqrt{5}}{5}$
- $\tan\left(\frac{\theta}{2}\right) = -2$

55.
- $\sin(2\theta) = -\frac{120}{169}$
- $\cos(2\theta) = \frac{119}{169}$
- $\tan(2\theta) = -\frac{120}{119}$
- $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{26}}{26}$
- $\cos\left(\frac{\theta}{2}\right) = -\frac{5\sqrt{26}}{26}$
- $\tan\left(\frac{\theta}{2}\right) = -\frac{1}{5}$

56.
- $\sin(2\theta) = -\frac{120}{169}$
- $\cos(2\theta) = \frac{119}{169}$
- $\tan(2\theta) = -\frac{120}{119}$
- $\sin\left(\frac{\theta}{2}\right) = \frac{5\sqrt{26}}{26}$
- $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{26}}{26}$
- $\tan\left(\frac{\theta}{2}\right) = 5$

57.
- $\sin(2\theta) = -\frac{4}{5}$
- $\cos(2\theta) = -\frac{3}{5}$
- $\tan(2\theta) = \frac{4}{3}$
- $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{50-10\sqrt{5}}}{10}$
- $\cos\left(\frac{\theta}{2}\right) = -\frac{\sqrt{50+10\sqrt{5}}}{10}$
- $\tan\left(\frac{\theta}{2}\right) = -\sqrt{\frac{5-\sqrt{5}}{5+\sqrt{5}}}$
  $\tan\left(\frac{\theta}{2}\right) = \frac{5-5\sqrt{5}}{10}$

58.
- $\sin(2\theta) = -\frac{4}{5}$
- $\cos(2\theta) = -\frac{3}{5}$
- $\tan(2\theta) = \frac{4}{3}$
- $\sin\left(\frac{\theta}{2}\right) = \frac{\sqrt{50+10\sqrt{5}}}{10}$
- $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{50-10\sqrt{5}}}{10}$
- $\tan\left(\frac{\theta}{2}\right) = \sqrt{\frac{5+\sqrt{5}}{5-\sqrt{5}}}$
  $\tan\left(\frac{\theta}{2}\right) = \frac{5+5\sqrt{5}}{10}$

74. $\frac{\cos(2\theta)+\cos(8\theta)}{2}$

75. $\frac{\cos(5\theta)-\cos(9\theta)}{2}$

76. $\frac{\sin(8\theta)+\sin(10\theta)}{2}$

77. $\frac{\cos(4\theta)+\cos(8\theta)}{2}$

78. $\frac{\cos(\theta)-\cos(5\theta)}{2}$

79. $\frac{\sin(2\theta)+\sin(4\theta)}{2}$

80. $2\cos(4\theta)\cos(\theta)$

81. $-2\cos\left(\frac{9}{2}\theta\right)\sin\left(\frac{5}{2}\theta\right)$

82. $2\sin\left(\frac{11}{2}\theta\right)\sin\left(\frac{1}{2}\theta\right)$

83. $2\cos(4\theta)\sin(5\theta)$

84. $\sqrt{2}\cos\left(\theta-\frac{\pi}{4}\right)$

85. $-\sqrt{2}\sin\left(\theta-\frac{\pi}{4}\right)$

90. $1-\frac{x^2}{2}$

91. $\frac{14x}{x^2+49}$

92. $\ln|x+\sqrt{x^2+16}|-\ln(4)$